Plane-wave imaging using synthetic aperture imaging reconstruction technique with regularized singular-value decomposition (RSVD)

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Abstract— **Plane wave ultrasound imaging (PW) with coherent compounding can improve the image quality in terms of contrast, SNR, and lateral resolution compared to the conventional PW method. When the number of PW transmissions is small, the image quality still need improvement. In this paper, we proposed to first estimate the synthetic transmit aperture imaging (STA) data with regularized singular-value decomposition (RSVD) from the PW RF data, and then used the STA reconstruction method to reconstruct ultrasound images. Compared to the delay-and-sum, the contrast-to-noise ratio (CNR) values of the hypo-echoic inclusions in both simulation and experiment was improved approximately by 28% and 19%, respectively, while the spatial resolutions were similar in the proposed method. The objects at the central column in the field of view have image metrics comparable to those of the full data set (75 PW emissions)**

Keywords—synthetic aperture reconstruction, pseudoinverse, regularized SVD

I. INTRODUCTION

Ultrasound imaging based on transmitting a single plane wave (PW) with parallel receive beamforming techniques enables a very high ultrafast frame rates (higher than 5000 fps) [1]-[3]. Such an ultrafast technique was used to capture the tissue motion information by introducing shear mechanical waves to propagate into the human soft tissues, in order to estimate the local tissue viscoelastic properties [4]. However, it suffers from poor image qualities in terms of spatial resolution and contrast due to the single PW transmission [5], [6]. To overcome this limitation, Montaldo *et al.* [7] proposed to coherently add the ultrasonic images obtained from different titled PW transmission angles. When the number of PW transmissions is small, the image qualities still need improvement. Coherent plane wave compounded imaging has strong conceptual similarities with the synthetic aperture method [8], [9]. In synthetic transmit aperture imaging (STA) [10], each element in transducer array is excited consecutively and RF backscatter signals are acquired by all the receiving channels. Each transmit-and-receive RF echo would be used to reconstruct a low resolution image using delay-and-sum (DAS) algorithm. Afterwards, all the low resolution images can be coherently combined to produce a high resolution image. Dynamic focusing in both transmit and receive provides ultrasound images with optimal spatial resolution [10].

In this paper, we proposed to apply the synthetic aperture imaging reconstruction technique to plane-wave imaging to improve the image qualities in the case of a small number of transmissions. From the PW RF data, we first estimated the synthetic transmit aperture imaging data in the frequency domain with a regularized singular-value decomposition (RSVD) method. In RSVD, a smooth filter was used to suppress the noise amplification caused by the small singular values. The estimated RF data spectrum is equivalent to conventional STA data in single-element transmission. Then, the time-domain estimated RF data were obtained using the inverse Fourier transform. Finally, the ultrasound images were reconstructed using the standard image reconstruction method in STA.

In Section II, the theory of the proposed method will be presented. The implementation of the method will be described. In Section III, the image qualities of the tested images from the proposed method will be assessed. The discussion and conclusion will be presented in Section IV.

II. METHODS

A. Synthetic aperture plane wave imaging

In the transmission process, PW can be viewed as a special case of delay-encoded transmission [11]. Generally, we assume there are N elements in the transducer array. In coherent plane wave compounding method, L plane-wave transmissions would be used in the RF data acquisition to form a high-resolution image. For each transmission with certain inclination angle α, a discrete amount delay d, is applied to each transmitting element n (n=1:N), which is mainly determined by the inclination angle. Therefore, a delay matrix **D** with the size of L-by-N can be defined. Each column of **D** represents the delay for each element at the corresponding transmission angles, and the row index represents the transmission order l (l = 1:L). By using **D**, a delay-encoded matrix **A** is constructed in a similar format as in [11] in Fourier domain as,

$$A_{ln}(f) = e^{-i2\pi f d_{ln}} \tag{1}$$

where f is temporal frequency in the spectrum, **A** is the matrix with elements of $A_{ln}(f)$. d_{ln}, the element in **D**, is the delay for the n-th transmit element at the l-th transmission.

In each transmission, $M(t)$, is the plane wave RF signal, where $M_{lk}(t)$ is the signal at t received by the k-th (k = 1:K) receiver in the l-th transmission. $M(t)$ can be acquired by that the RF signal $S(t)$, which is the equivalent traditional STA signal. $S_{nk}(t)$ is the signal transmitted at n-th element, and received by k-th receiving element. Therefore, in the frequency domain, the encoding process for generating the plane wave RF data at each frequency can be expressed as,

$$A_{ln}(f)S_{nk}(f) = M_{lk}(f),$$

or,

$$\mathbf{AS} = \mathbf{M}, \tag{2}$$

where **A**, **S** and **M** is the matrix with elements of $A_{ln}(f)$, $S_{nk}(f)$, $M_{lk}(f)$, respectively. Therefore, the equivalent STA signal can be estimated by solving Eq. (2). Using matrix **S**, RF data of synthetic aperture imaging can be calculated by an inverse Fourier transform of **S**, and a high-resolution image can be obtained by summing low-resolution images formed from the traditional STA imaging data.

B. Decoding process for estimating STA RF signal

The goal of this decoding process is to estimate **S** from **M** stably since **M** usually contains the noise. In addition, when the number of transmissions and receivers is smaller than the number of elements ($L, K<N$), the matrix **A** is underdetermined. Thus, pseudoinverse with regularization method is necessary to be used to estimate the equivalent traditional STA signal.

In this paper, we used SVD as the decoding operation [12]. By using SVD, the matrix **A** can be decomposed into three matrices, **U**, **Σ**, **V**:

$$\mathbf{A} = \mathbf{U\Sigma V}^T, \tag{3}$$

where **U** and **V** are two orthogonal matrices, **Σ** is a diagonal matrix, and the superscript T means the transpose of the matrix. Therefore the matrix **S** can be estimated as

$$\mathbf{S} = \mathbf{V\Sigma^{-1}U}^T\,\mathbf{M}, \tag{4}$$

where **Σ** is a diagonal matrix with the element of singular values [13]. In this study, **A** is underdetermined. Therefore, $\mathbf{\Sigma^{-1}}$ does not exist. In addition, the small singular values might amplify the noise in **M**. Therefore, we implemented a regularized SVD method with a smooth filter for each temporal frequency, similar to the truncated SVD [14], in the singular value domain to filter out the noise caused by the small singular values. It is worth noting that, this filter is frequency dependent. The RSVD filter for each frequency was designed as [14]

$$f_m = \frac{\sigma_m^2}{\sigma_m^2 + w^2}, \tag{5}$$

where σ_m (m = 1:r, r is the rank of **A**) is the singular values in matrix **Σ** and w is a regularized factor. w can be adjusted until the optimized image quality is obtained. In this paper, we chose w as $\sigma_1/2$, where σ_1 is the first and largest singular value for each frequency.

C. Implemetation method

This proposed synthetic aperture plane wave (SAPW) imaging method was applied to all the four tested phantoms provided by the Plane-wave Imaging Challenge in Medical UltraSound (PICMUS) in the 2016 IEEE International Ultrasonics Symposium. The proposed method was in the category of 11 PWs, which were selected at angles ranging from -2.16 degrees to 2.16 degrees in a step size of 0.43 degree.

III. RESULTS

Fig. 1 shows the log-enveloped beamformed images obtained with the proposed method for the four challenge datasets. Compared to the reference images, the mean CNR values of the hypo-inclusions in both simulation and experiment have been improved by approximately 28% and 19%, respectively. Moreover, the spatial resolution was preserved in all tested phantoms. The detail quatification results for the tested phantoms are showing in TABLE I.

TABLE I. Quantification results of the simulation and experimental phantoms.

Contrast Speckle	CNR	Resolution Distortion	Axial/Lateral resolution (mm)
Experimental	11.40	Experimental	0.57/0.87
Simulation	14.69	Simulation	0.41/0.64

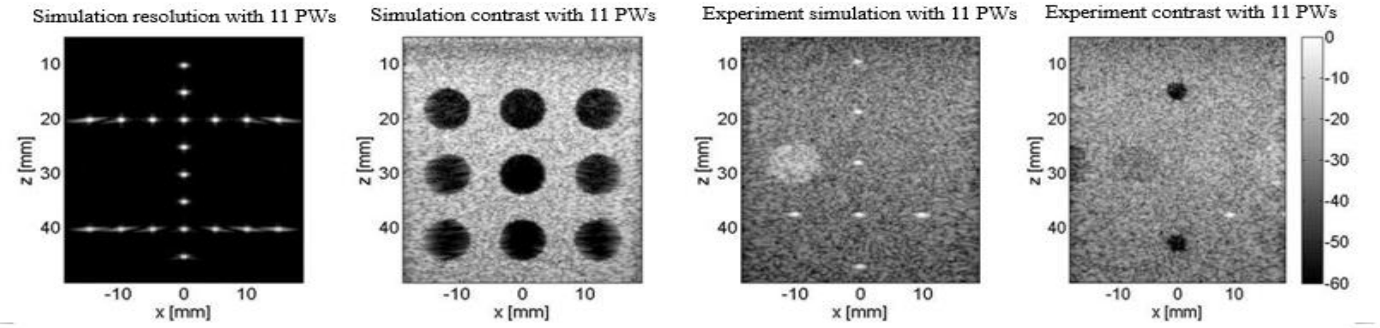

Fig.1. Log-enveloped beamformed images obtained from proposed method for the four challenge datasets

IV. SUMMARY AND CONCLUSION

The estimated SAPW RF signal obtained from the proposed method was stabilized by suppressing the negative effect caused by the smallest singular values. The regularized factor in the frequency-dependent designed filter was chosen as the half of the first singular value, by trial and error. Further investigations in choosing different regularized factors is needed to optimize the image quality.

Compared to the 11 plane waves at the same angle range, the results obtained from the proposed method demonstrated the improved CNR for simulation and experimental phantoms. The spatial resolution can also be preserved. The objects at the central column of the field of view were reconstructed with image metrics comparable to these of the full dataset (75 PWs). However, the image quality for the targets located at both sides of the images still need to be improved.

In PW image reconstruction, it was assumed that an infinite extent PW is emitted from the array probe to the target region. However, because of the finite size of the ultrasound array, this assumption is not valid for points that are located at the edge or outside of the ultrasound beam. The method proposed in this paper can reconstruct images without assuming infinite extent of PWs. Therefore, it can potentially improve the image qualities of PW imaging, especially for deep targets. This is because the targets at large depth will be out of the plane wave beam even at a small steering angle of PWs. In addition, apodization functions can be applied in both the transmit and receive aperture in STA image reconstruction, which can further improve the image contrast over the standard plane wave reconstruction algorithm.

In conclusion, this paper presents the use of a synthetic aperture imaging reconstruction technique for the plane-wave imaging. Combining with the frequency-dependent regularized filter, the plane wave transmissions can be used to estimate the traditional STA RF signal in single-element transmission. The reconstructed images obtained from the standard STA image reconstruction method can improve the image quality when the number of PW emissions is small.

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REFERENCES

[1] L. Sandrin, S. Catheline, M. Tanter, X. Hennequin, and M. Fink, “Time resolved pulsed elastography with ultrafast ultrasonic imaging,” *Ultrason. Imaging*, vol. 21, pp. 259–272, Oct. 1999.

[2] L. Sandrin, S. Catheline, M. Tanter, C. Vinconneau, and M. Fink, “2D transient elastography,” *Acoust. Imaging*, vol. 25, pp. 485–492, Jan. 2000.

[3] L. Sandrin, M. Tanter, S. Catheline, and M. Fink, “Shear modulus imaging using 2D transient elastography,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 49, no. 4, pp. 426–435, Apr. 2002.

[4] M. Tanter and M. Fink, “Ultrafast Imaing in Biomedical Ultrasound’, IEEE, Trans., Ultraso. Ferrpelectr. Freq. Contol, vol. 61, No.1, Jan, 2014

[5] J. Bercoff, M. Tanter, and M. Fink, “Sonic boom in soft materials: The elastic Cerenkov effect,” *Appl. Phys. Lett.*, vol. 84, no. 12, pp. 2202–2204, Mar. 2004.

[6] J. Bercoff, M. Tanter, and M. Fink, “Supersonic shear imaging: A new technique for soft tissues elasticity mapping,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 51, no. 4, pp. 396–409, Apr. 2004.

[7] G. Montaldo, M. Tanter, J. Bercoff, N. Benech, and M. Fink, “Coherent planewave compounding for very high frame rate ultrasonography and transient elastography.” *IEEE Transactions on Ultrason. Ferroelectr. Freq. Control*, vol. 56, pp. 489–506, 2009.

[8] J. A. Jensen, O. Holm, L. J. Jerisen, H. Bendsen, S. I. Nikolov, B. G. Tomov, P. Munk, M. Hansen, K. Salomonsen, J. Hansen, K. Gormsen, H. M. Pedersen, and K. L. Gammelmark, “Ultrasound research scanner for real-time synthetic aperture data acquisition,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 52, no. 5, pp. 881–891, May 2005.

[9] T. Misaridis and J. A. Jensen, “Use of modulated excitation signals in medical ultrasound. Part III: High frame rate imaging,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 52, no. 2, pp. 208– 219, Feb. 2005.

[10] J. A. Jensen, S. I. Nikolov, K. L. Lokke, and M. H. Pedersen, “Synthetic aperture ultrasound imaging”, *Ultrasonics*, vol. 44, pp.e5-e15, 2006.

[11] P. Gong, M. C. Kolios and Y. Xu, "Delay-encoded transmission and image reconstruction method in synthetic transmit aperture imaging, "*IEEE transactions on ultrasonics, ferroelectrics, and frequency control,* vol. 62, pp. 1745-56, 2015.

[12] A. N. Tikhonov, “Solution of incorrectly formulated problems and the regularization method” Sov. Math. Dokl., 5, pp, 1035-1038, 1963.

[13] J. M. Varah, “A practical examination of some numerical methods for linear discrete ill-posed problems”, SIAM Review, 21, pp. 100-111. 1979.

[14] P. C. Hansen, “The truncated SVD as a method for regularization,” BIT vol. 27, pp. 543-553, 1987.